Equilibrium Risk Pools in a Regulated Market with Costly Capital

Stephen Mildenhall

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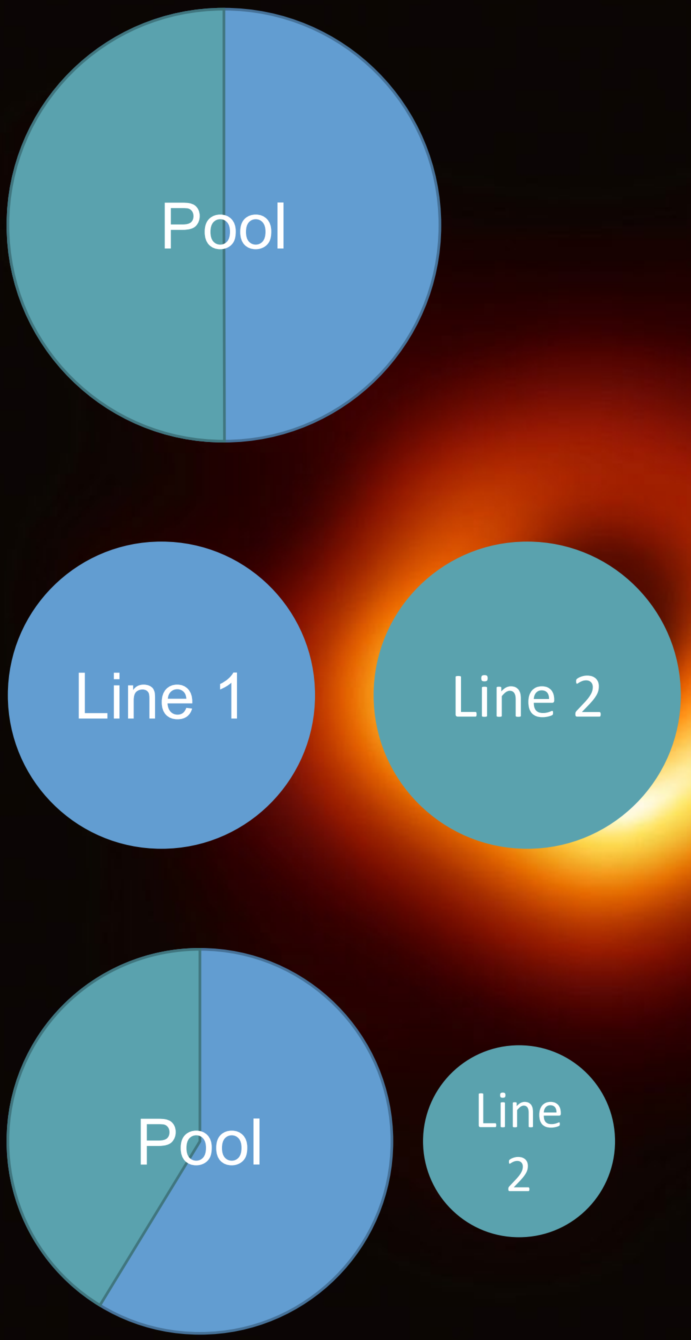
Three Minute Summary

- Theoretical attraction of diversification...but wonder
- why are there so many small insurers, often writing volatile classes?
- Determine conditions that imply one risk pool is optimal
- How diversification benefit is shared?
- Analyze using a two-line model, with different pricing and regulatory capital assumptions



Big black hole:
Single pool insurer

Gravity repels:
Separate insurers

Standard

Partial attraction:
Pool and monoline

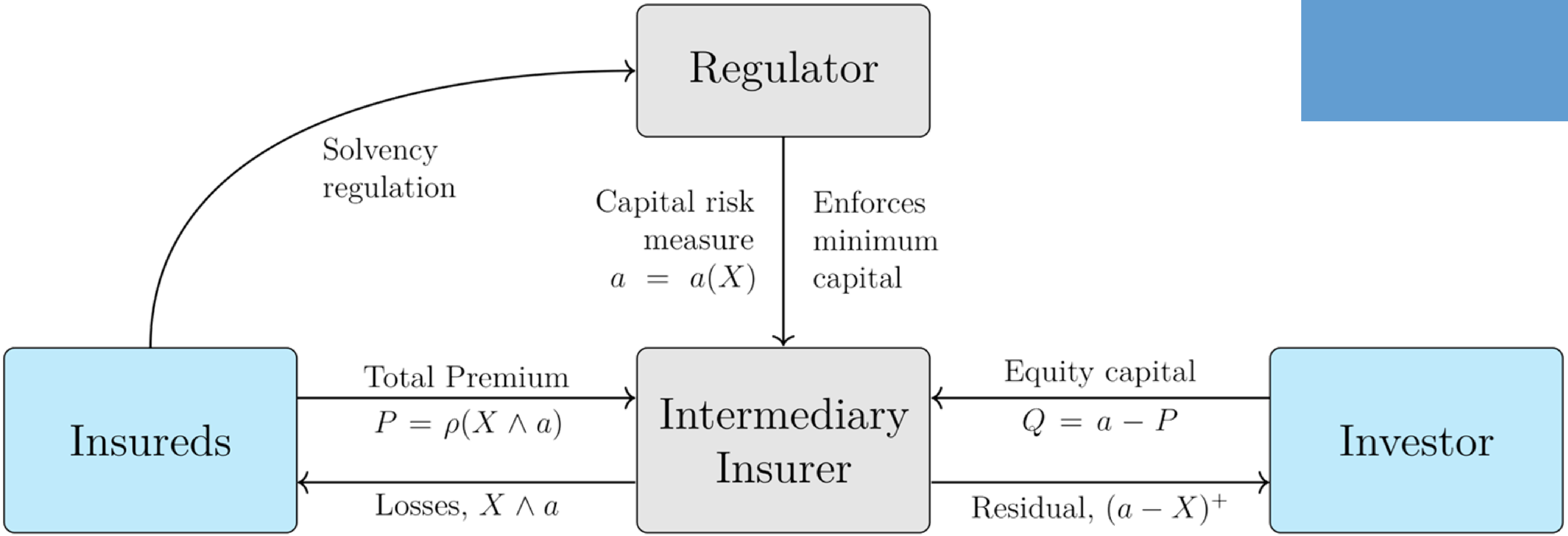
New & interesting

convex risk

Context and Literature

- Capital allocation and multiline pricing: ex post equal priority default rule, perfect markets, with frictional costs of holding capital, Cummins (RMIR 2000)
  – Phillips, Cummins, Allen (JRI 1998), Myers, Read (JRI 2001), Sherris (JRI 2006), Ibragimov, Jaffee, Walden (JRI 2010)
- We assuming imperfect market but no frictional costs of capital: opposite of literature
- Risk neutral, ambiguity averse investors, who charge for shape of risk using a non-additive distortion pricing functional
  – Wang (ASTIN 1996), Wang, Young, Panjer (IME 1997)
- Even though pricing is non-additive it is consistent with general equilibrium and no arbitrage
  – De Waegenaere, Kast, and Lapied (IME 2003), Chateauneuf, Kast, Lapied (Math Fin 1996)
  – Bid-ask spread, Castagnoli, Maccheroni, Marinacci, (Math Fin 2004)
- Gravity repels solution = diversification traps: Ibragimov, Walden (2007) applies with very thick tails
- Ibragimov, Jaffee, Walden (Rev Fin 2018): perfect market with frictional costs

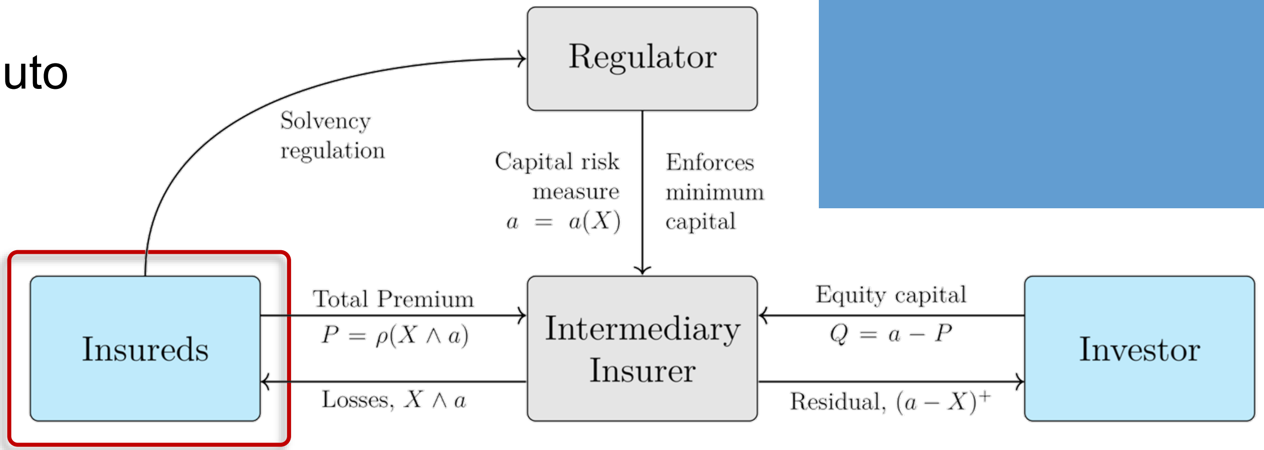
Four Actors and Their Interactions



- One-period model, no expenses, no investment income, no taxes; risk transfer and not risk pooling

Insured Loss Distributions

- Two classes (lines) of insured
  - Low-risk class: high frequency, low severity; US auto
  - High-risk class: catastrophe exposed property



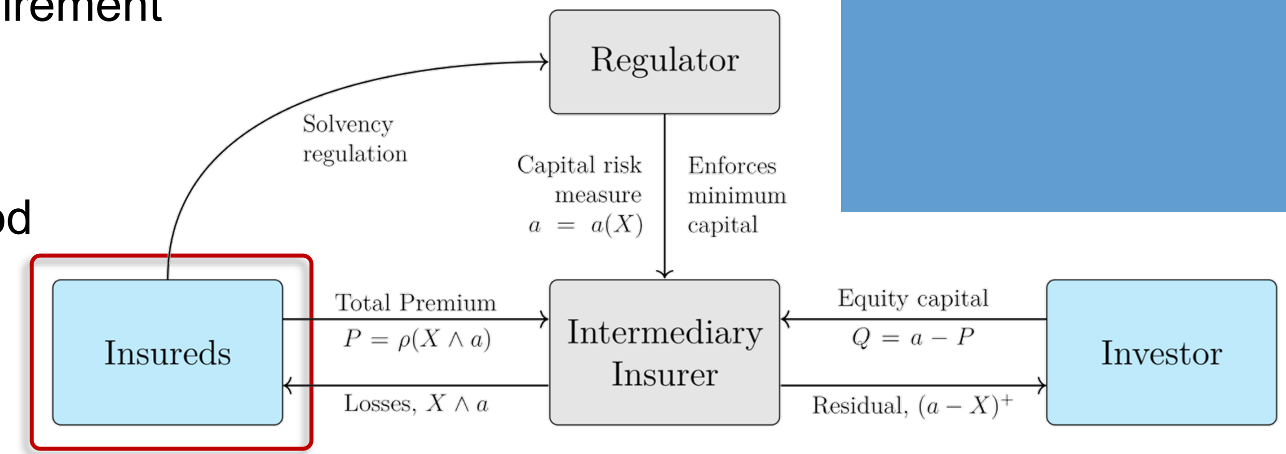
- Risk is a characteristic of class and not the individual insured

- Homogeneous loss model (Phillips, Myers Read,...)
  - Results for a sub-pool of a class are proportional to the results for whole class, model loss ratio
  - Pool size is not a consideration
  - Realistic beyond smallest portfolios and for catastrophe-exposed lines; Boonen, Tsanakas, Wuthrich (IME 2016); Mildenhall (Risks, 2017)

- Low risk class X_0, and relatively higher risk class X_1

Insured Buying Behavior

- Face mandatory / quasi-mandatory insurance requirement
  - Financial responsibility laws for auto
  - Workers compensation
  - Collateral protection: homeowners, property, flood
  - Contract: surety, GL
  - 60% of premium (Aon Benfield, 2015)



- Mandate is for third-party protection
  - Single policy form that satisfies insurance requirement
  - Insureds do not care about insurer solvency, provided policy satisfies mandatory requirement
  - Insureds judgment proof or guarantee funds

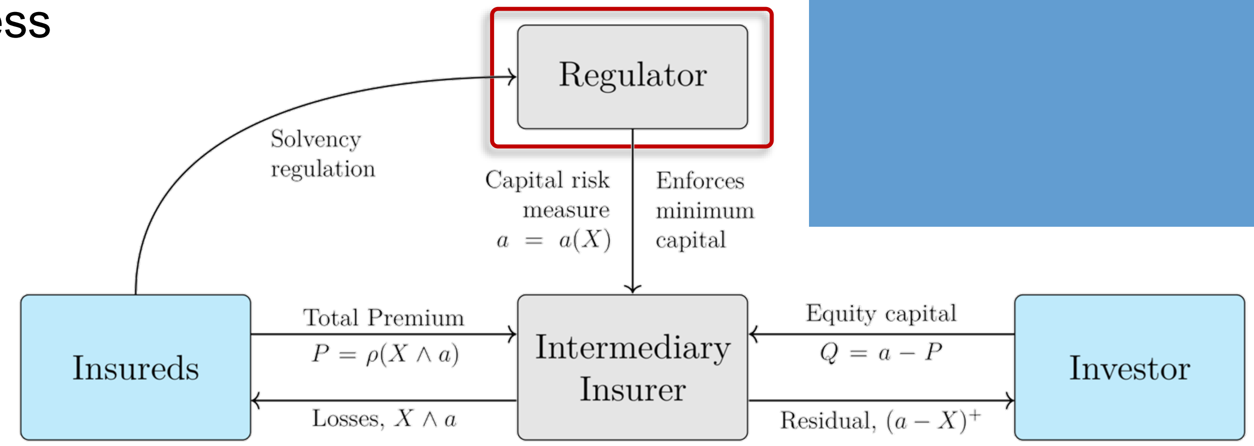
- Insureds are pure price buyers

convex risk

Regulator

- Solvency regulation necessary to ensure effectiveness of mandatory insurance
  - Risk-based capital standard
  - US NAIC RBC, Solvency II MCR, SCR, rating agency models

- Regulatory capital standard risk functional a = a(X) = a(total risk)
  - Homogeneous, monotone, translation invariant
  - Value at Risk (VaR) or tail value at risk
  - We use VaR in all examples



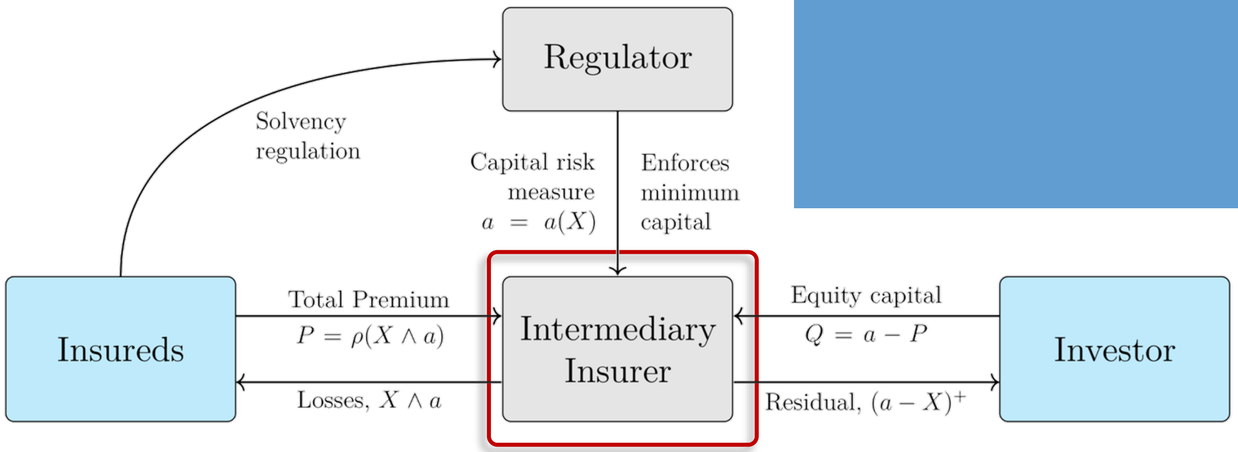
Incorporeal: regulator is a formula

- No other regulation beyond capital standard
  - Pricing based on agreed subjective probabilities and the investor ρ; catastrophe model
  - Any risk pool allowed provided it meets the capital standard: single policy to all whole market

Intermediary Insurer or Pool

- "Smart contract" incorporeal insurer or risk pool
  - Legal, organizational artefact
  - Pools exist to enable limited liability
  - Pools make economically meaningful adjustments to insurance payments in default states
  - Risk passed through to investors
  - Pooling lower ambiguity and lower cost
  - Vs. perfect market models have no role for pools

- No frictional cost for investor to hold assets in insurer
  - No transaction costs, no taxes
  - No management: no principle-agent problems
  - Minimal regulation, no trapped capital
  - Like a multi-insured catastrophe bond



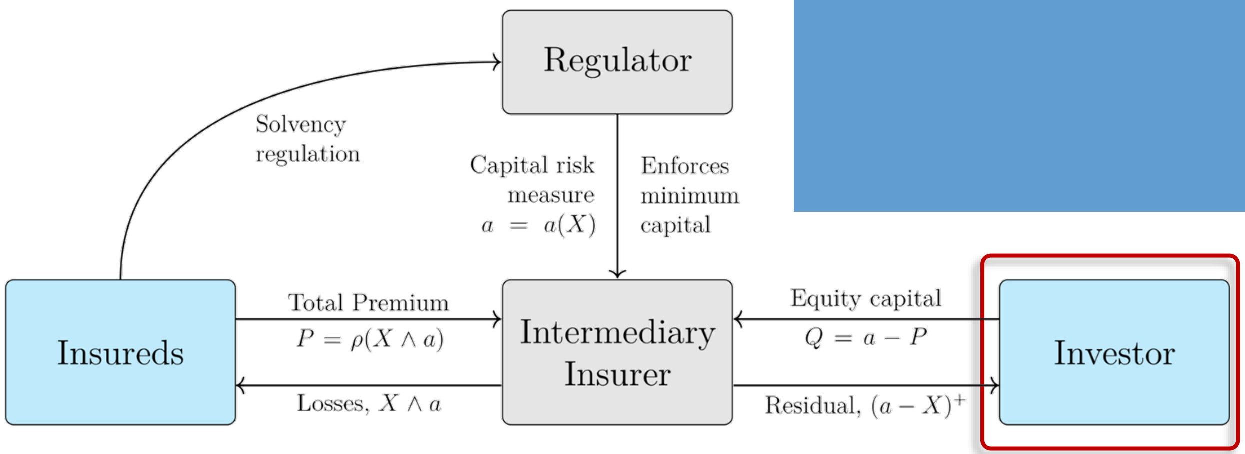
Incorporeal: insurer is a formula

Investor: Ultimate Risk Bearer

- Ambiguity averse but not necessarily risk averse

- Subjective probabilities

- Price the shape (distribution) of risk
  - Shape drives **risk**: standard deviation, VaR etc.
  - Shape drives **ambiguity**: 100-year event more ambiguous than personal auto



- Investors price using a **distortion risk measure** ρ, which prices any distribution X as ρ(X)
  - DRMs are coherent: sub-additive and respect diversification
  - Monotone, translation invariant, positive homogeneous, convex
  - Law invariant and comonotonic additive
  - Weighted average of VaR, with increasing positive weights, or of TVaRs, Kusuoka (2001)
  - Controlled by **distortion** $g : [0,1] \to [0,1]$, $g(s)$ is price of binary insurance with probability of loss s

Distortion Risk Measure Magic

- DRMs give unique allocation of pool premium back to individual insureds
  - Delbean (Coherent Risk Measure Notes, 2000), Venter, Major, Kreps (ASTIN 2006), Tsanakas (various)
  - Major, Mildenhall (2020)

- Allocation uses a risk adjusted probability measure with density g'(S(X)), S is survival function of X
  - Allocation to X_i is $E[X_i\, g'(S(X)]$

- DRMs can be calibrated to market pricing and are practical to work with

Pool Formation and Model Decision Variable

- Monoline pools on the same class can merge by homogeneity: equal pricing and capital
- Only one multiline pool can exist in equilibrium
  - Premium depends on mix by class in the pool
  - Two pools with different mixes: one would have cheaper premium, destroying equilibrium
- Conclude: by scaling there are only three possible market structures
  - Full pooling: one insurer
  - Two monoline insurers
  - One multiline pool insurer and one monoline insurer
- Market defined by proportion t of risk class 1 in the pool, $0 \le t \le 1$, and

– $t = 0, 1$	two monoline pools
– $t = 0.5$	full pooling
– $0 < t < 0.5$	class 0 fully pooled, class 1 split between pool and monoline
– $0.5 < t < 1$	class 1 fully pooled, class 0 split between pool and monoline

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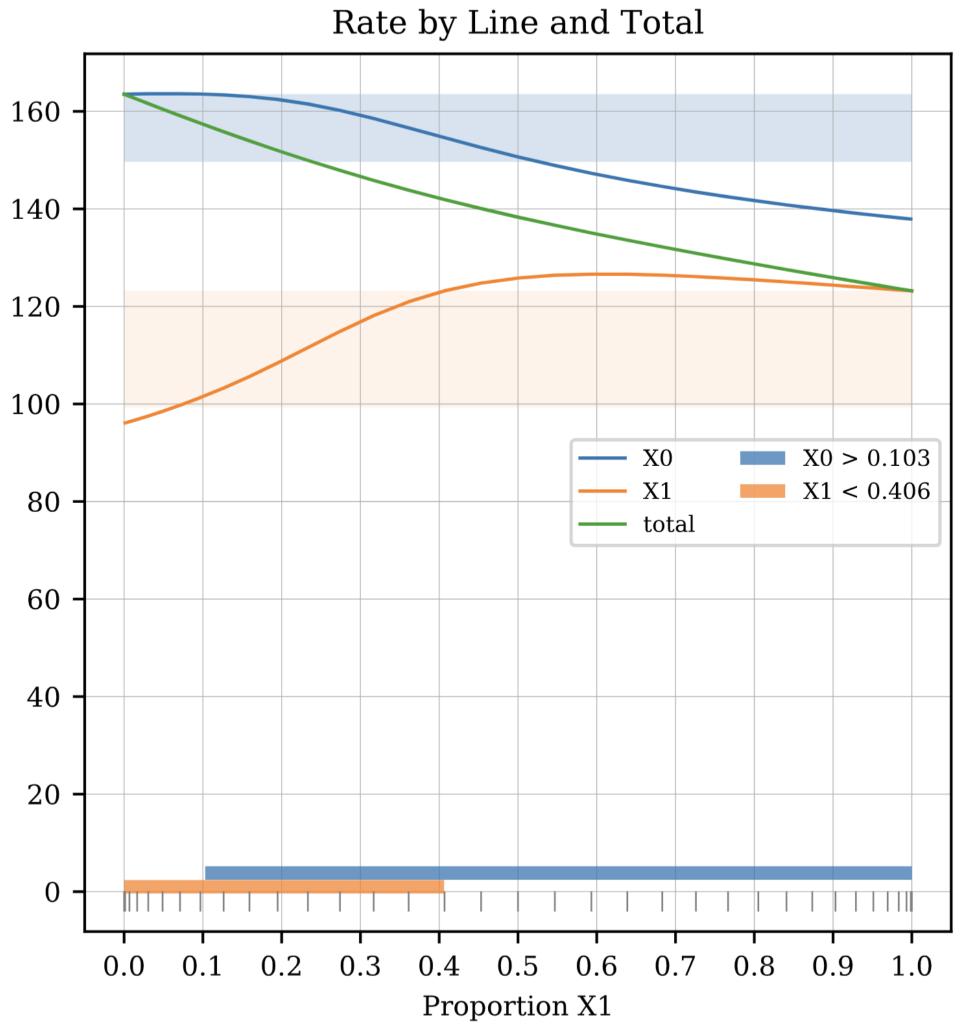
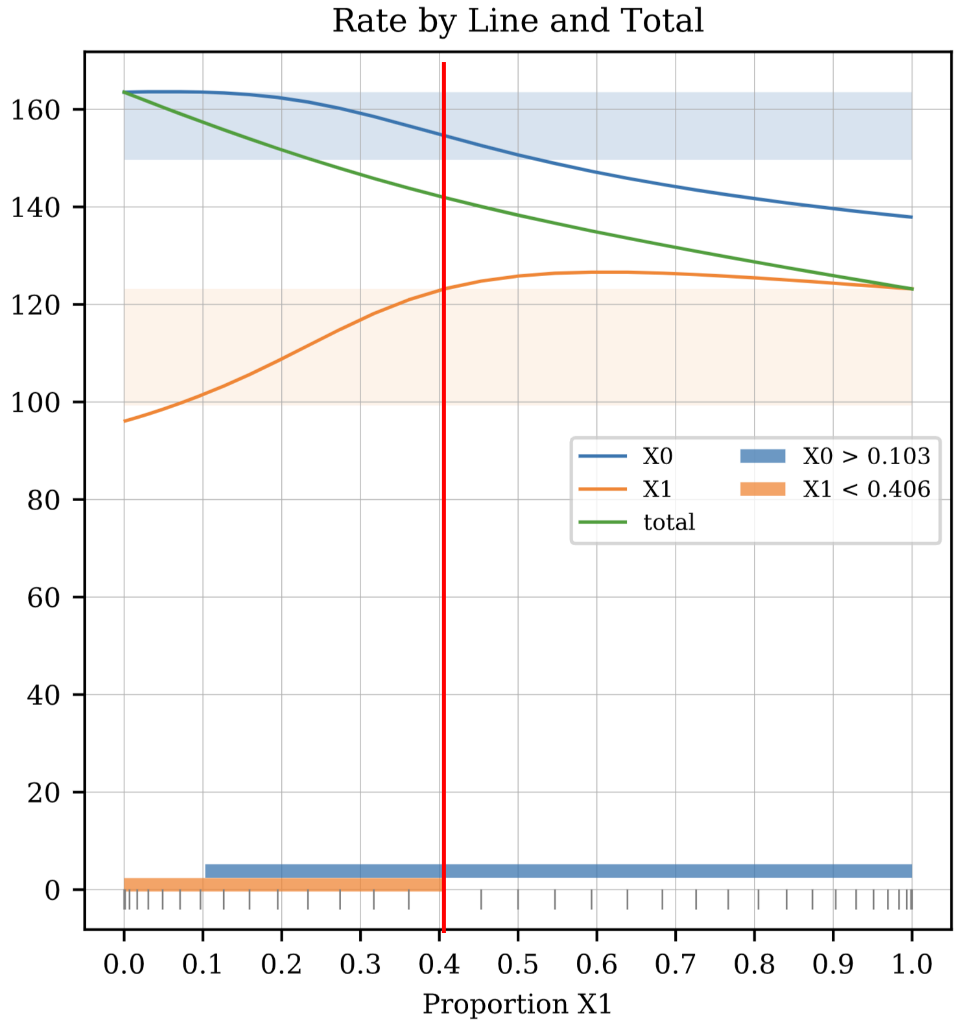
Example: Setup



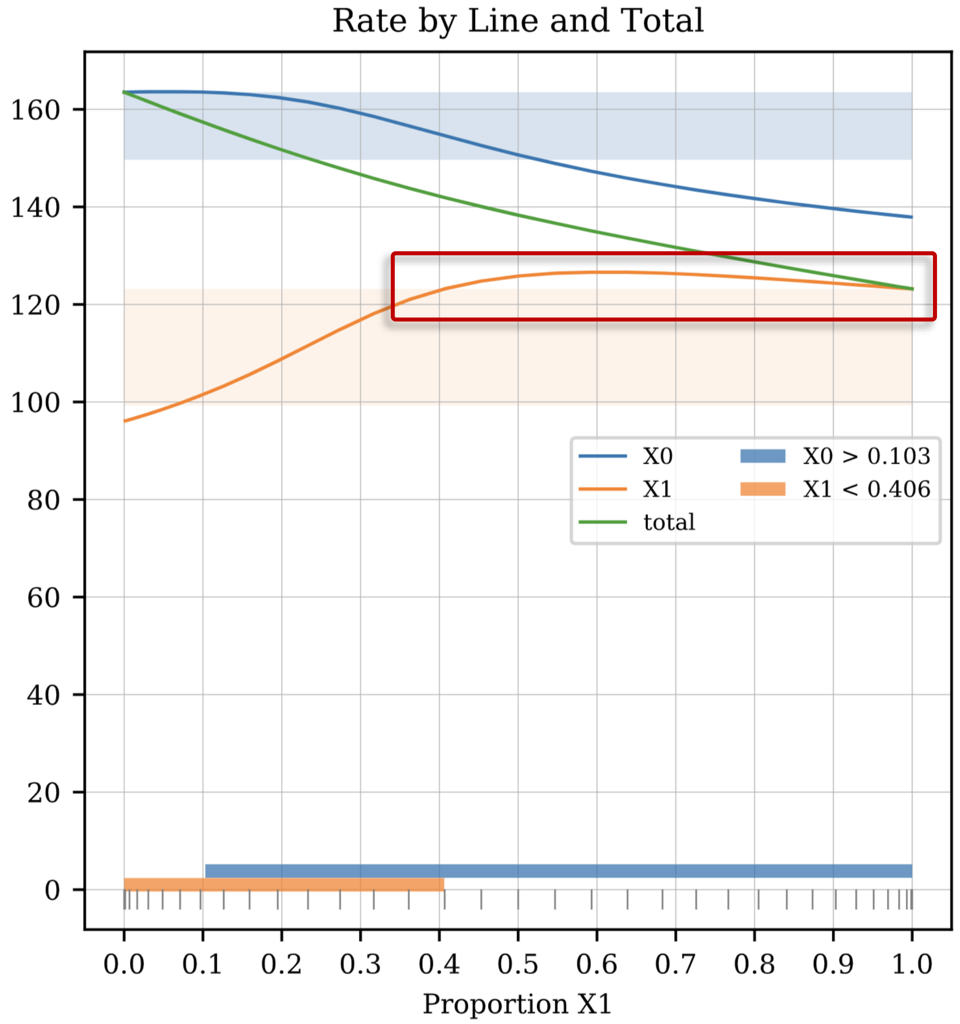
- *t*, the proportion of X_1, on x-axis
- Lines show **rate** for each line
  - Blue X_0 low, orange X_1 high risk
  - Green: blended pool rate
- Expected unlimited loss, before insurer default,
  - $X_0 = 150$
  - $X_1 = 100$
- Thin-tailed gamma distribution
  - X_0 CV 10%, e.g. US personal auto
  - X_1 CV 25%, e.g. commercial auto, WC
- Shaded bands at top show range from monoline loss cost and premium for each line
  - Orange X_1 thicker band because higher risk
- Expensive pricing, weak capital standard

Example: Partial Pooling Equilibrium Solution



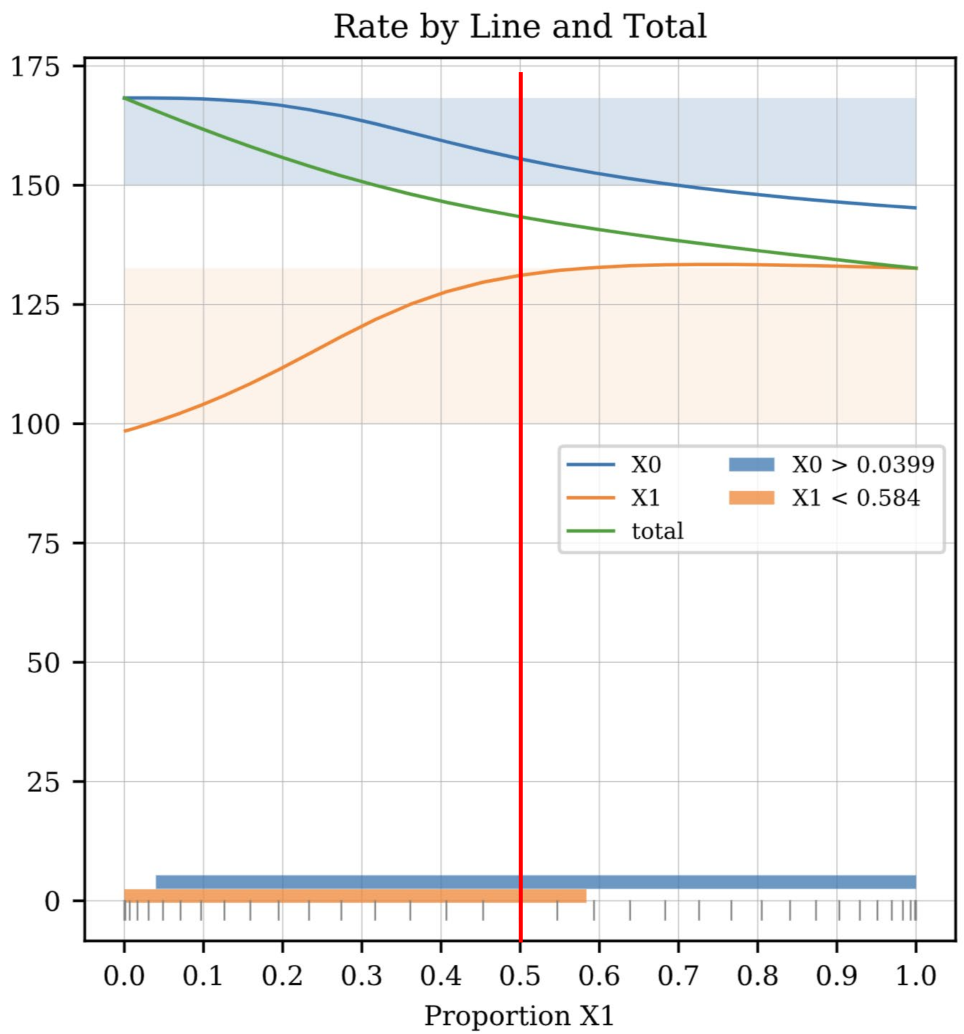
- Equilibrium solution
  - **X_0 and 2/3rds of X_1 are pooled; remaining 1/3rd of X_1 written monoline, *t* = 0.4**
- Why?
  - *t* > 0.4: X_1 rate greater than monoline...X_1 will not pool
  - *t* < 0.4: X_1 insureds in pool get below monoline rate, with remainder monoline
  - Remainder will offer to pool with X_0 at slightly higher rate until equilibrium reached at *t* = 0.4
  - **X_1 pays monoline rate and X_0 captures all diversification benefit**
  - Pareto optimal by shape of rate curves
  - 0.4 = 100 / 250: pool is equal expected loss mix of two lines; exact solution *t* = 0.406

Example: Rationale



- Why orange rate line bows up
- Adding small amount of X_0 to X_1 advantages X_1
  - Small amount of X_0 like adding a constant liability
  - X_1 thicker tailed...more likely to "cause" insolvency
  - ...by equal priority, proportion of liabilities, it picks up a greater share of assets in default
- Does not occur with unlimited capital
- Pooling increases the quality of insurance for X_1 and decreases it for X_0, relative to monoline
  - X_1 must pay economically fair pricing; greater than its monoline rate
  - X_0 pays less than monoline; in fact, here, less than monoline expected loss for *t* close to 1

Example: Full Pooling



- When $t = 0.5$ is feasible for both lines it is an equilibrium solution
- Why?
  - For $t \neq 0.5$ some insureds are forced into monoline rate, e.g.,
  - $t = 0.4$ some X_1 paying monoline rate would offer to pool with X_0 at $t = 0.45$ rate, bene-fitting them and X_0, original pool unravels
  - $t = 0.6$ some X_0 paying monoline rate would offer to pool with X_1 at t = 0.55 rate, benefitting them and X_1
  - At $t = 0.5$ all insureds pay lower multiline rate
  - No rational action can cause pool to unravel
- Diversification benefit shared more evenly
- Capital standard at Solvency II 99.5% level

convex risk

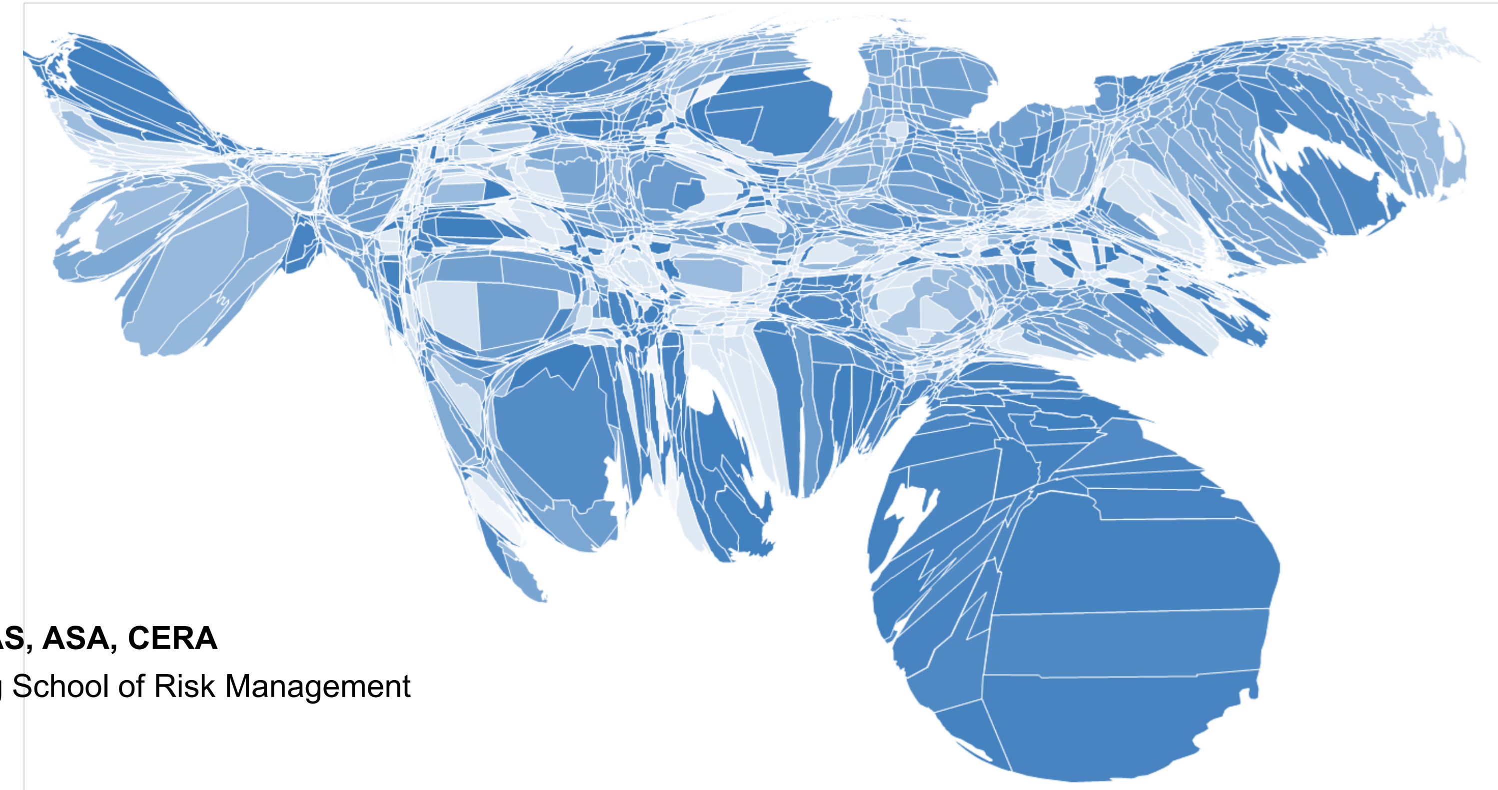
General Behavior and Conclusions

- Pooling solution determined by complex interaction between three variables
  - Relative tail thickness of X_0 and X_1
  - Strength of capital standard
  - Expense of insurance
- Full pooling is *more likely* with
  - Balanced tail thickness of the two lines
  - Stronger capital standard
  - More expensive insurance
- Two monoline pools occurs when regulatory risk measure is super-additive, thick tails
  - Green pool premium line bows up rather than down

Conclusions

- Market pricing functional, a combination of investor and regulator risk functionals, can fail to be sub-additive even when both components are coherent
- Diversification benefit of pooling is eroded by economic transfers caused by limited liability
- Weak capital standard can result in incomplete pooling and higher price for the riskier class
- Strong capital standard (almost) always results in full pooling

Contact Information



Stephen Mildenhall, PhD, FCAS, ASA, CERA
St. John's University, Greenberg School of Risk Management
Convex Risk LLC
New York, NY 100024
+1.312.961.8781 cell
mildenhs@stjohns.edu
steve@convexrisk.com

Graphic note: County size scaled to AAL estimates for hurricane, earthquake and severe weather using Gastner & Newman algorithm